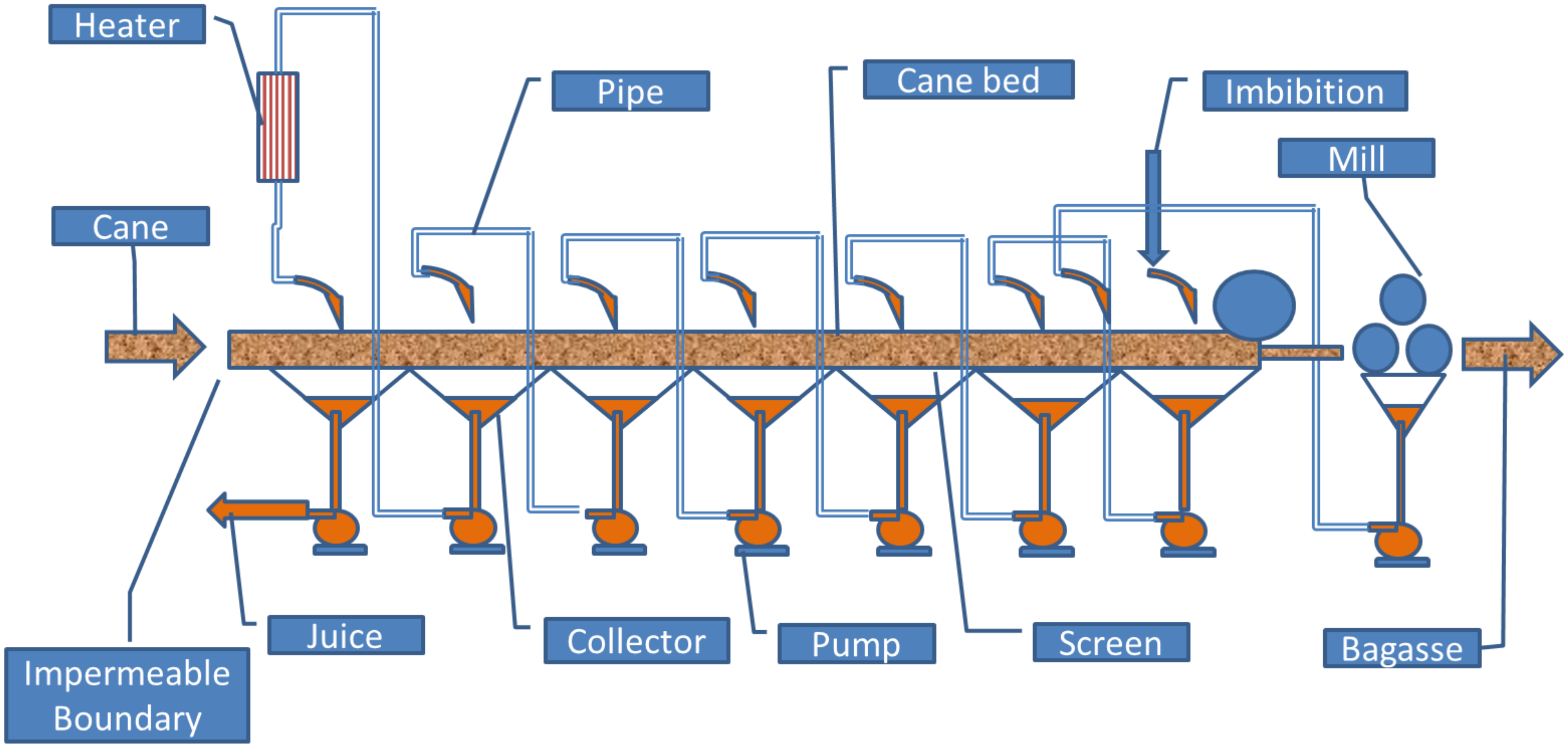
Juice holdup detection in a sugar cane diffuser

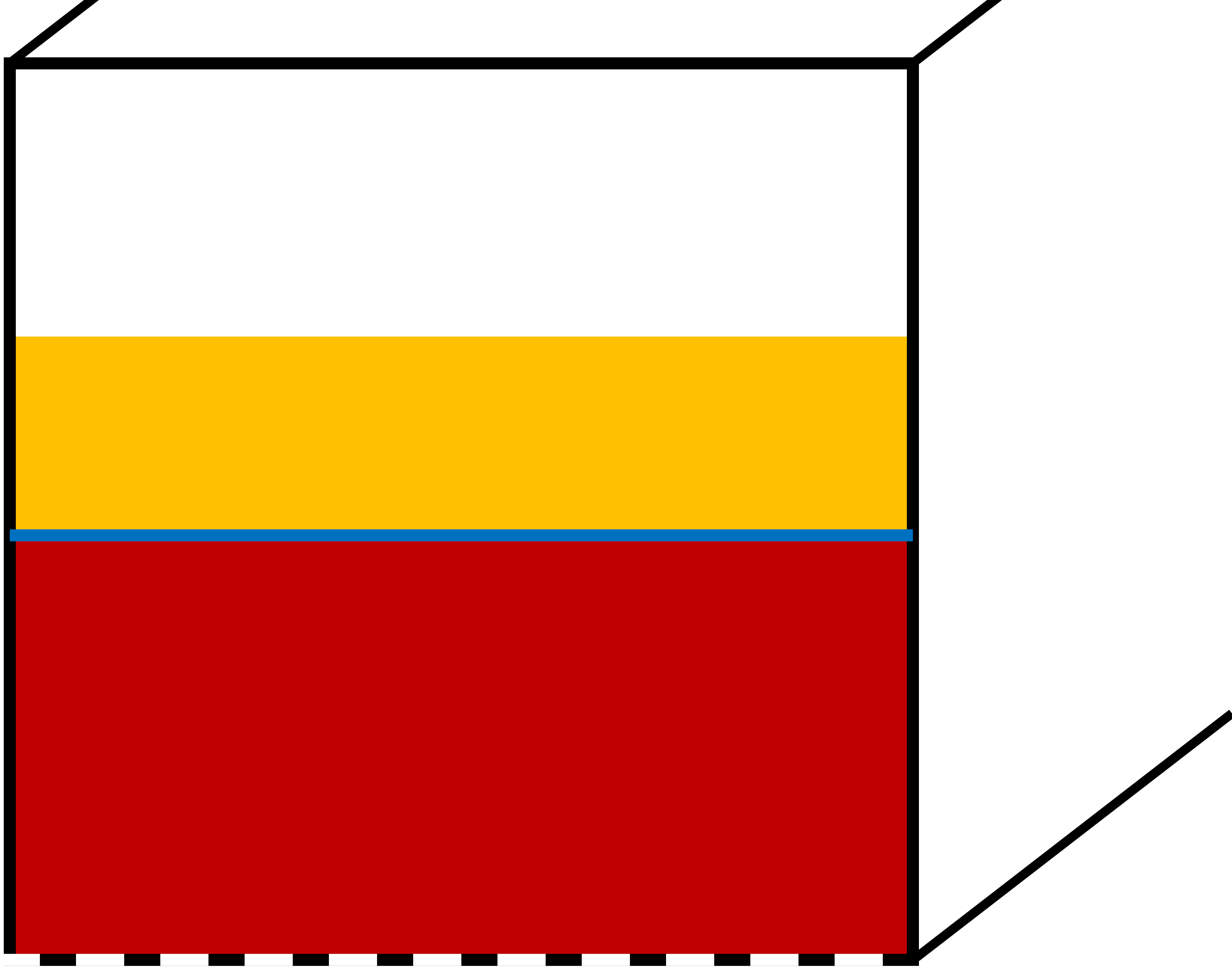
Oliver Whitehead, Stephen Wilson, Neville Fowkes

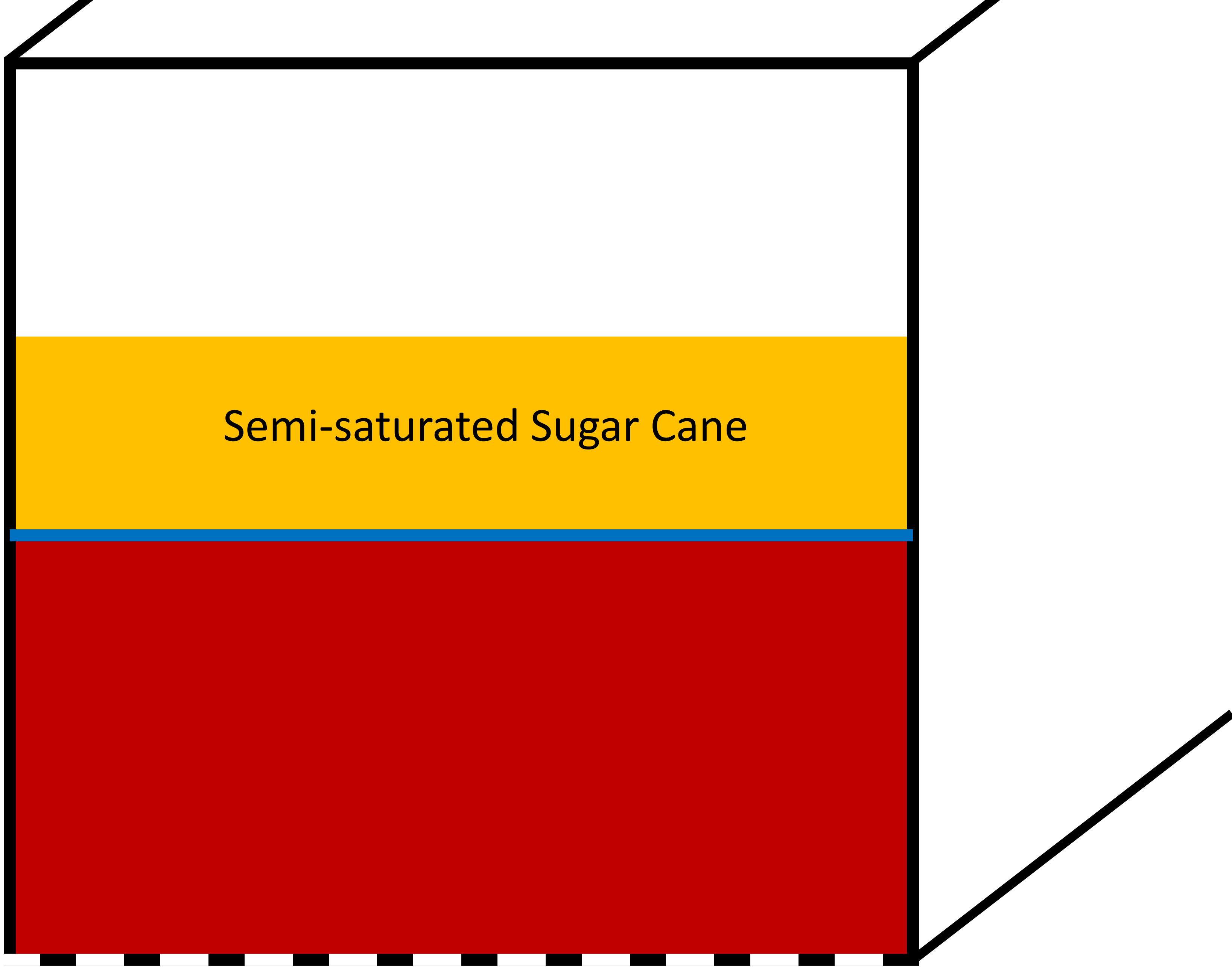
Industrial Representative: Richard Loubser

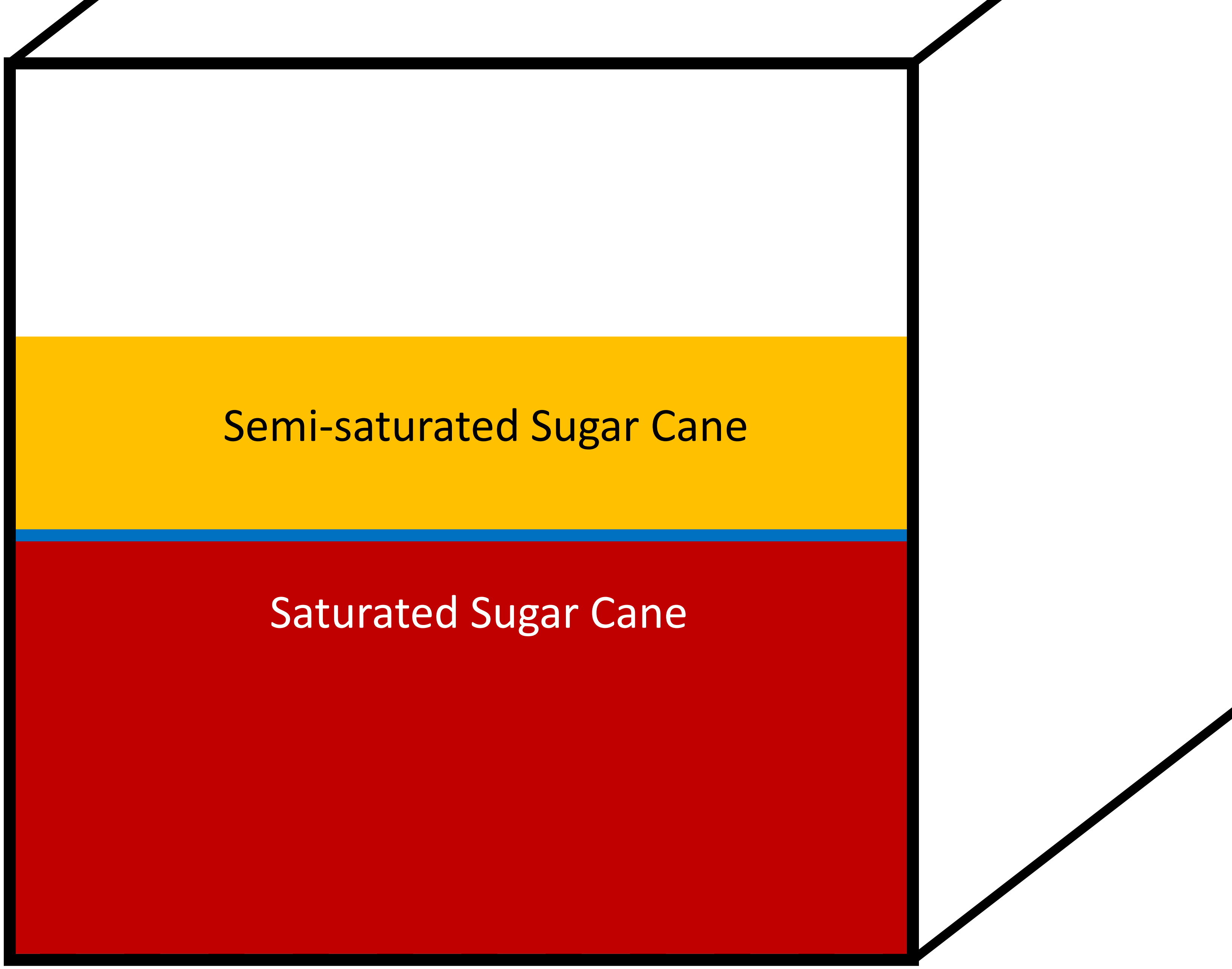
Sugar extraction with a diffuser



Diffuser




Semi-saturated Sugar Cane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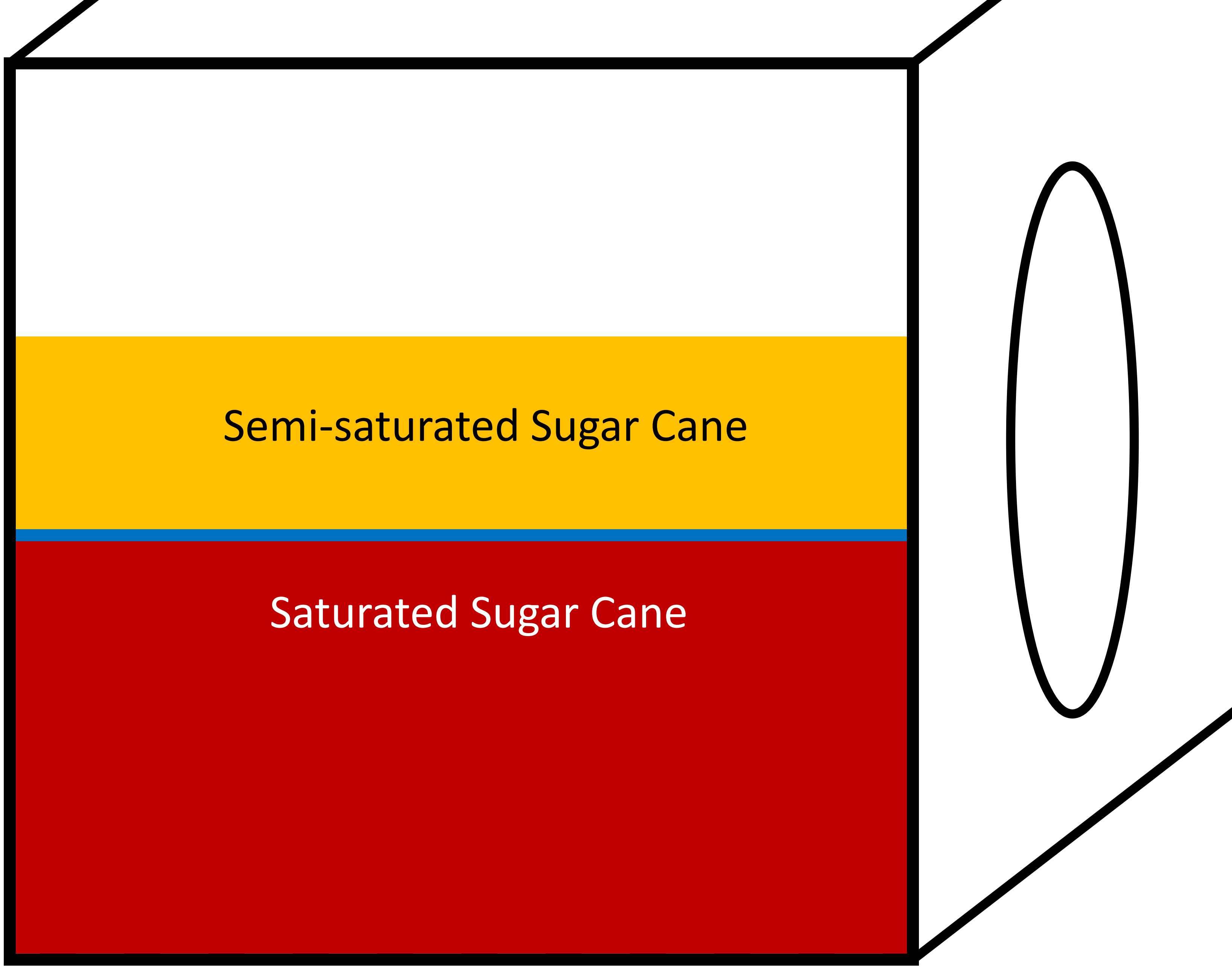
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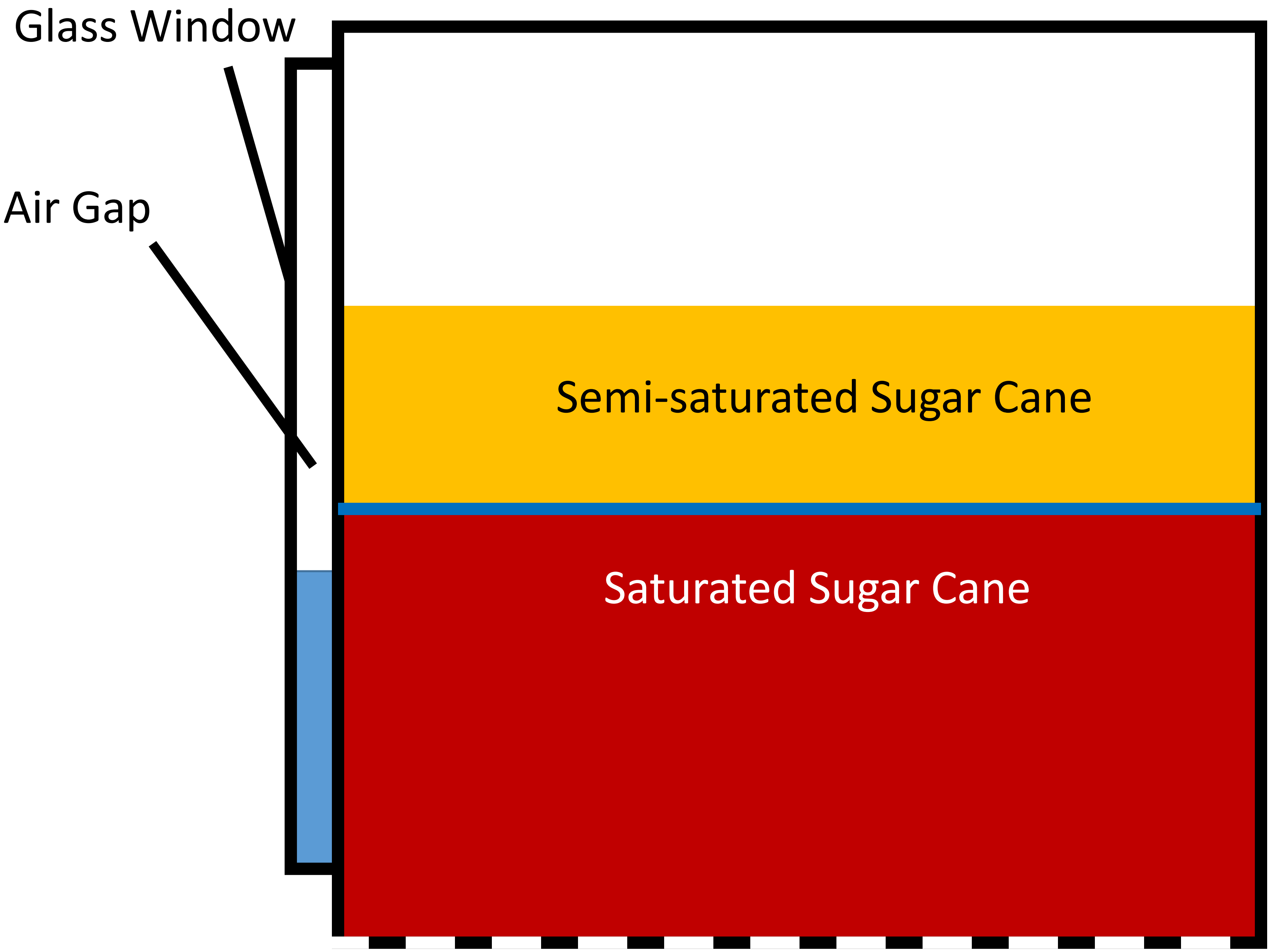
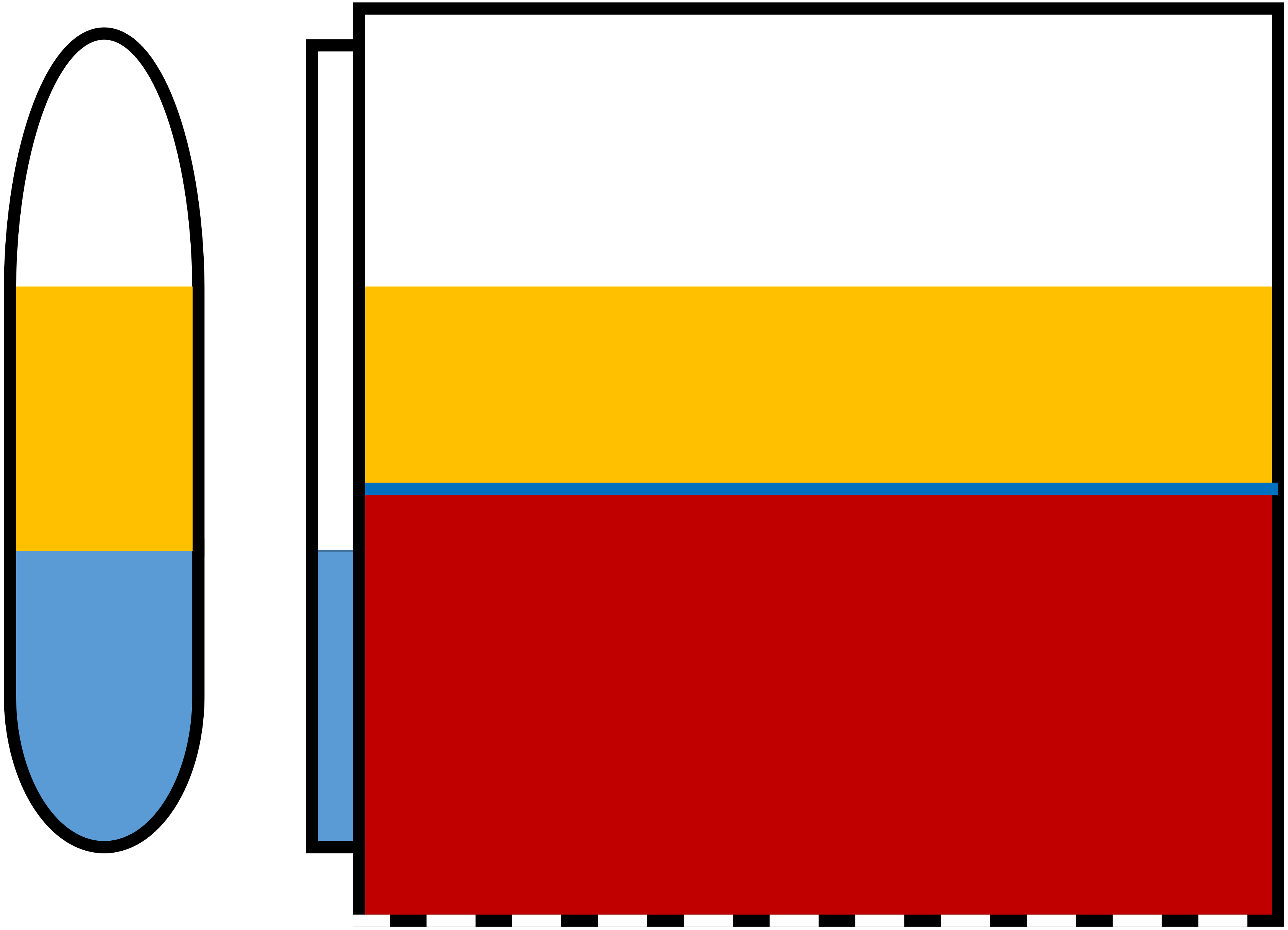
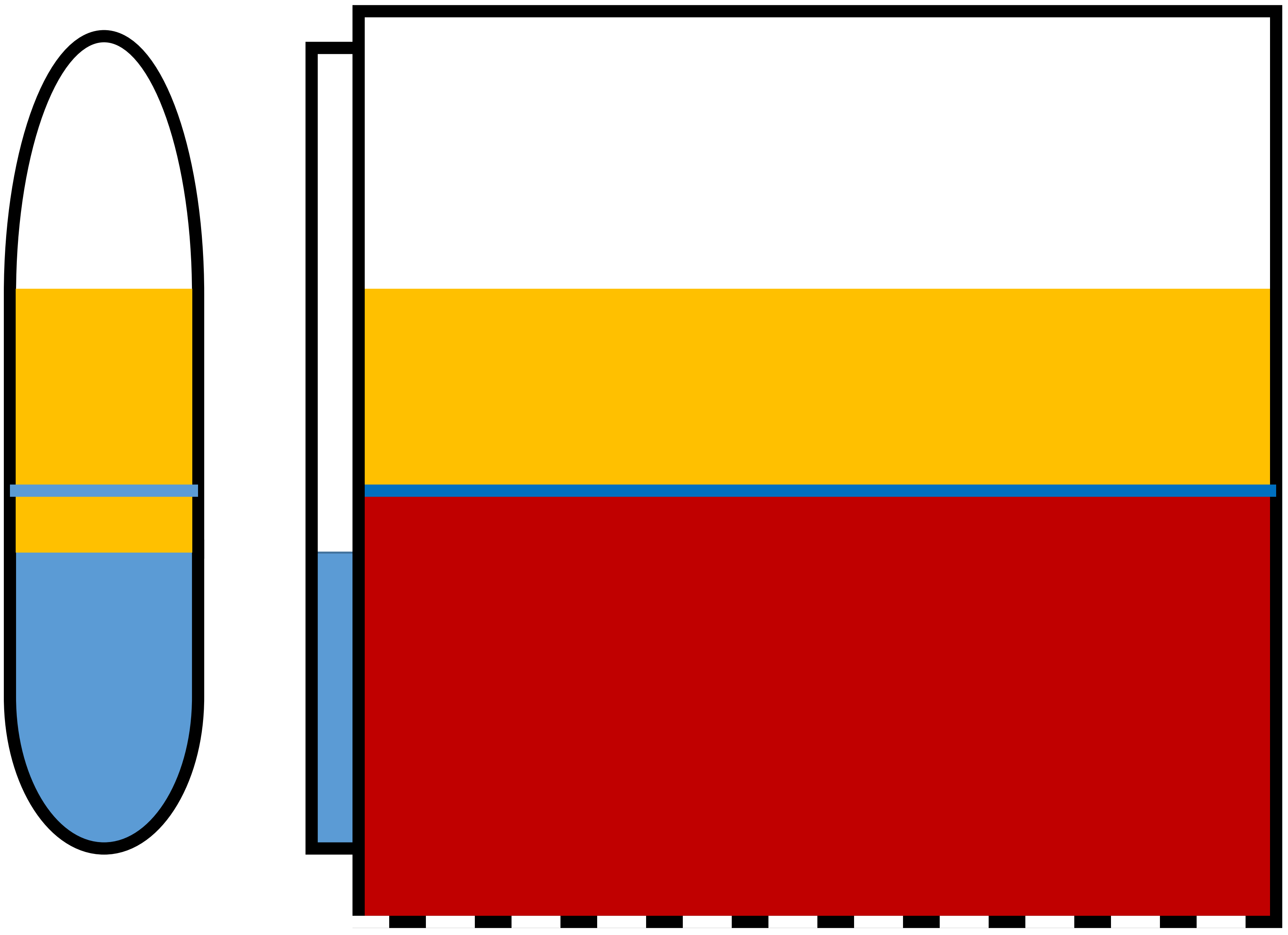
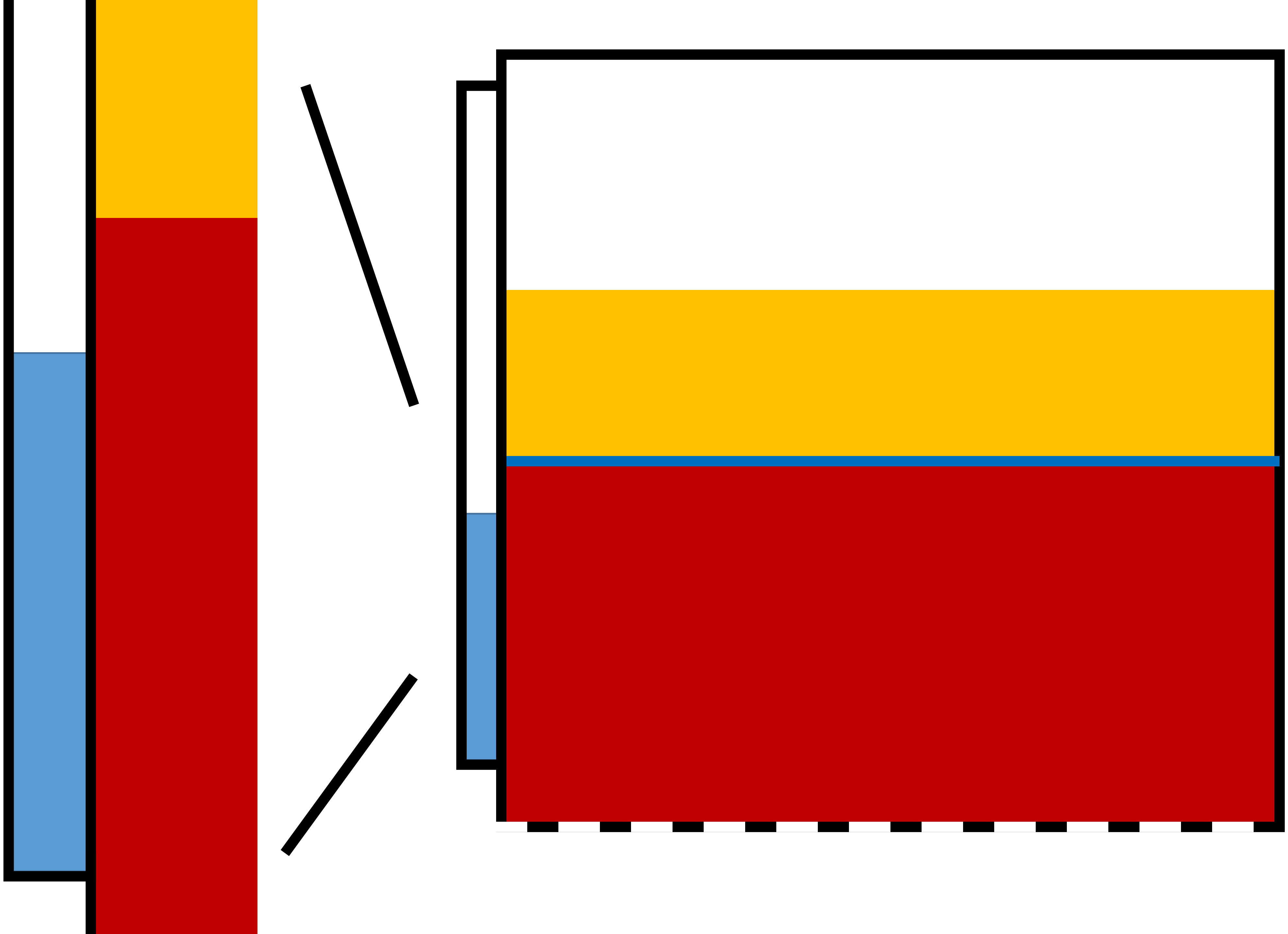
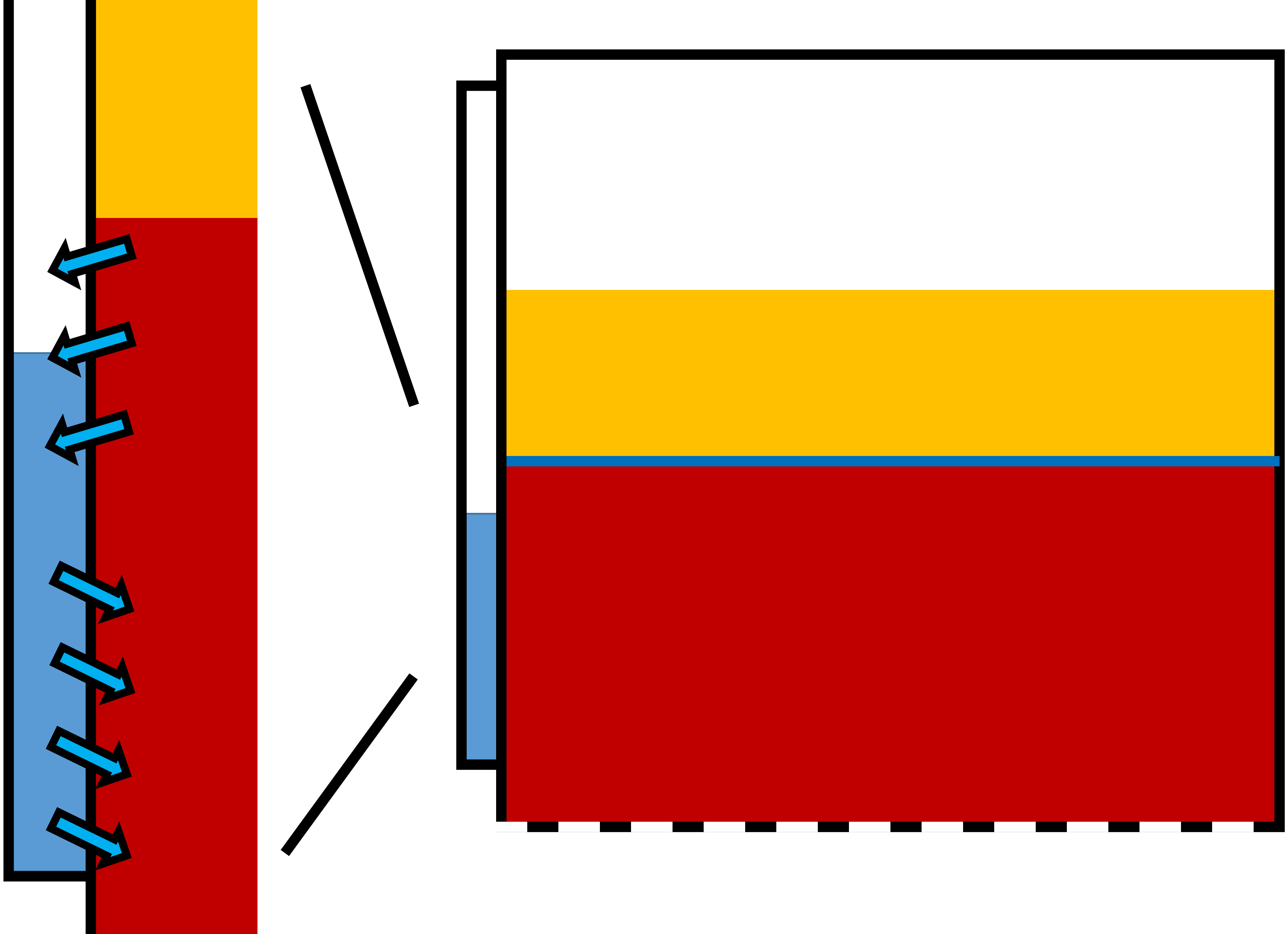
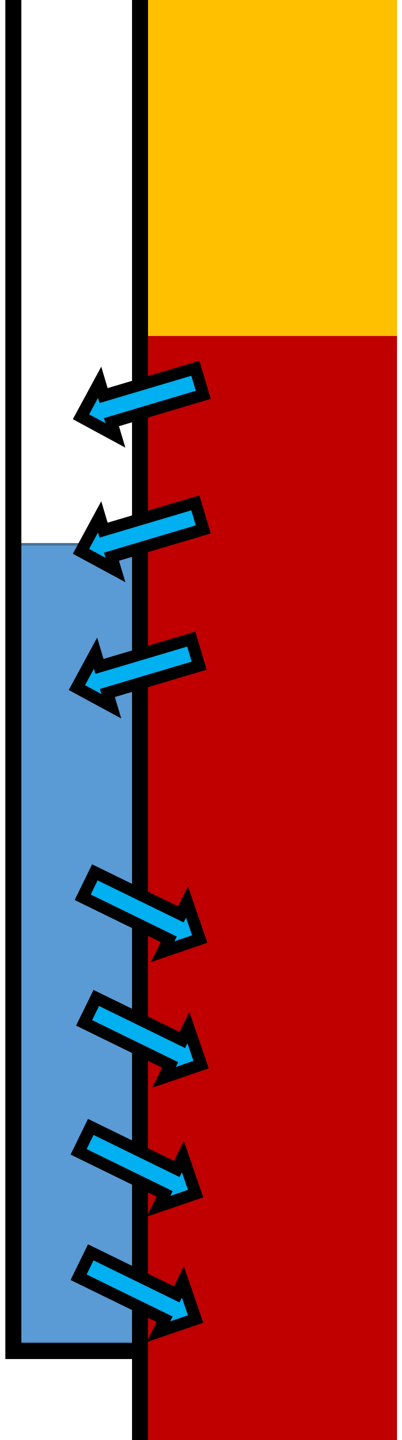

Glass Window
Air Gap
Semi-saturated Sugar Cane
Saturated Sugar Cane











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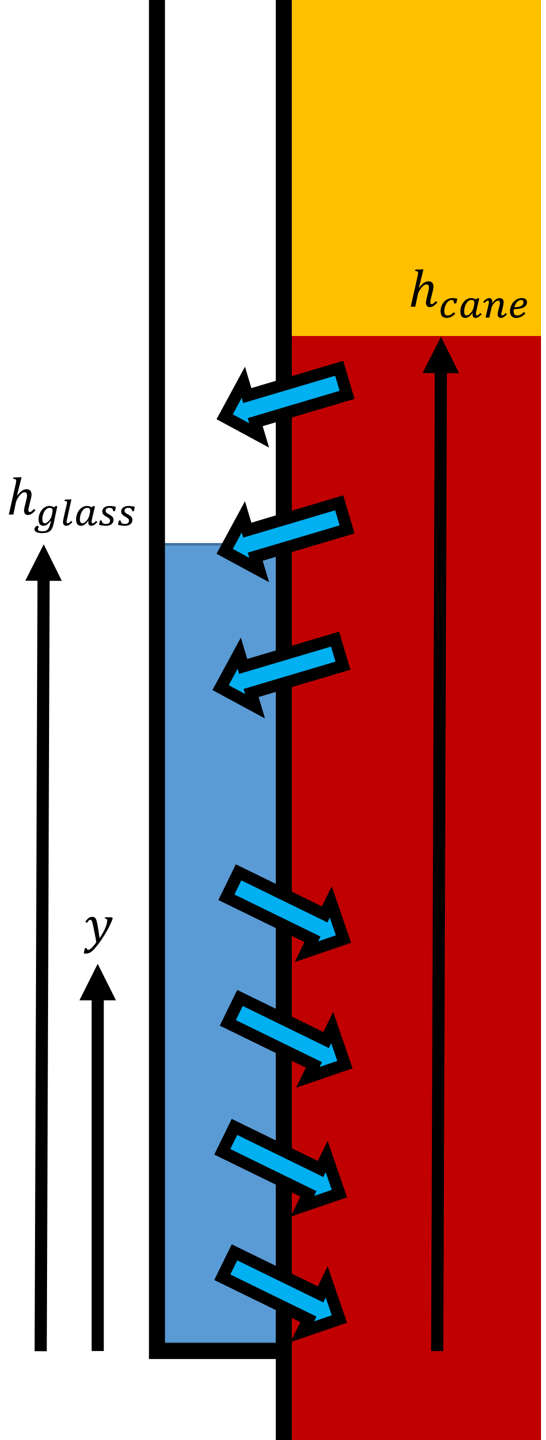
- Pressure in this glass gap and in the cane is purely hydrostatic



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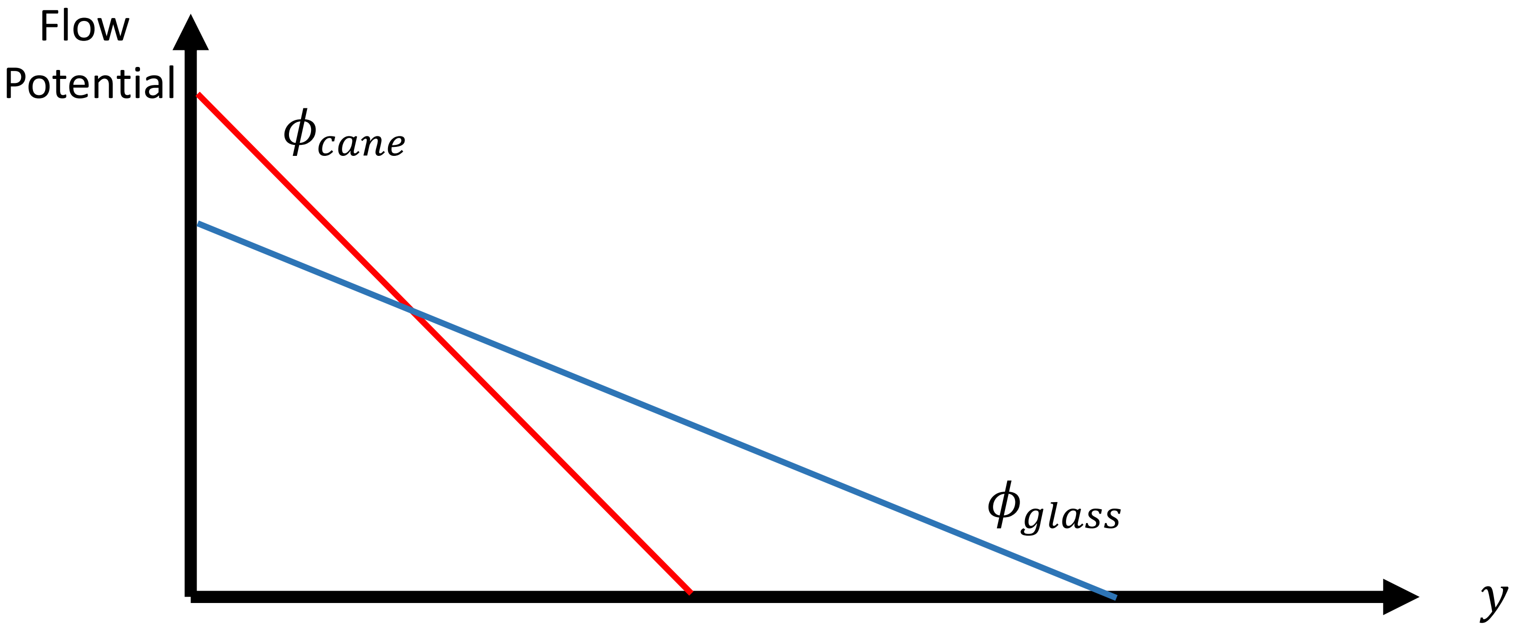

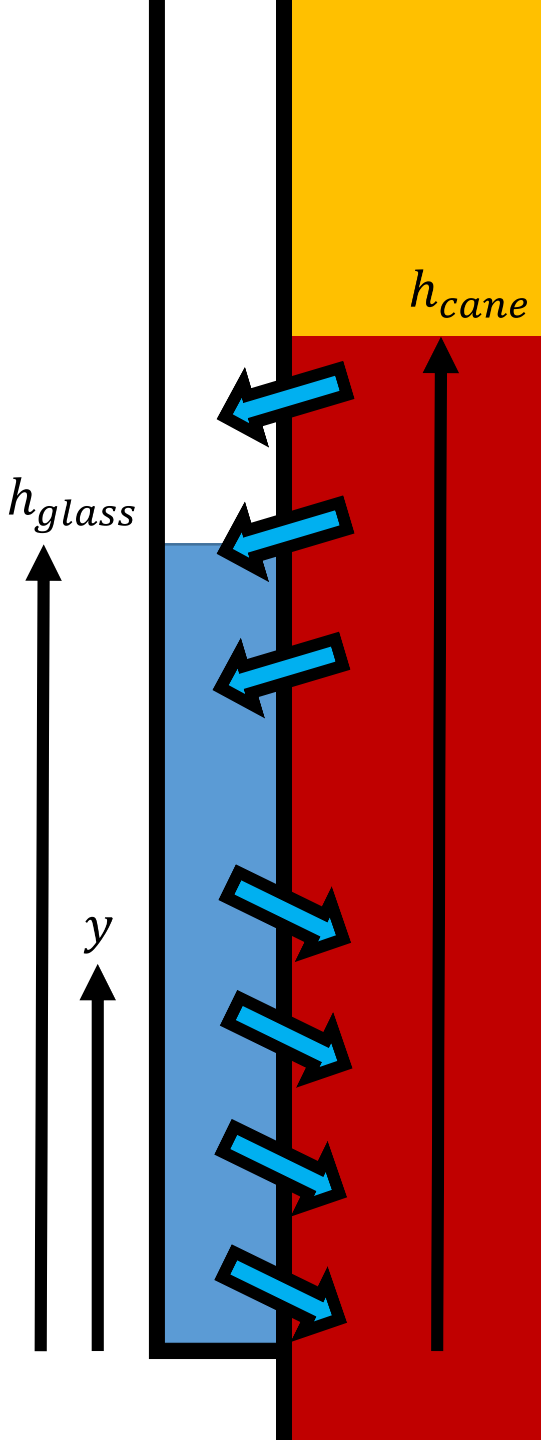

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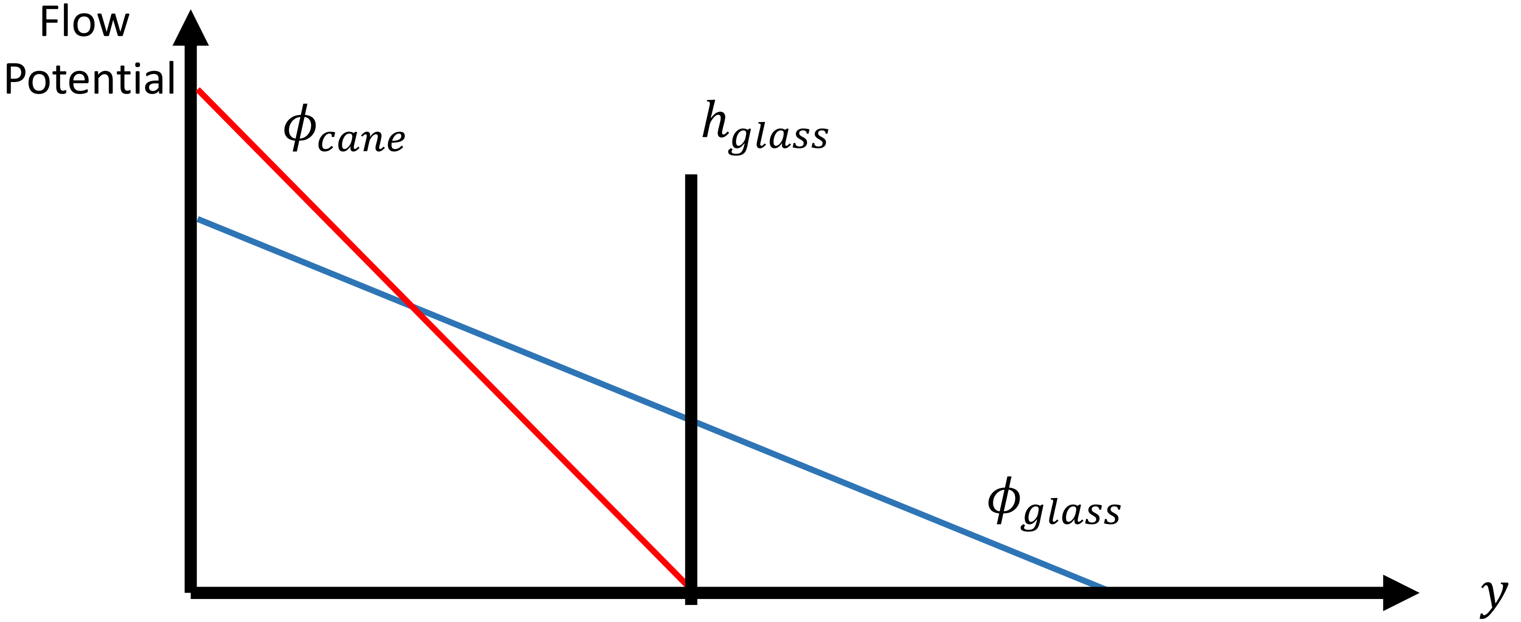

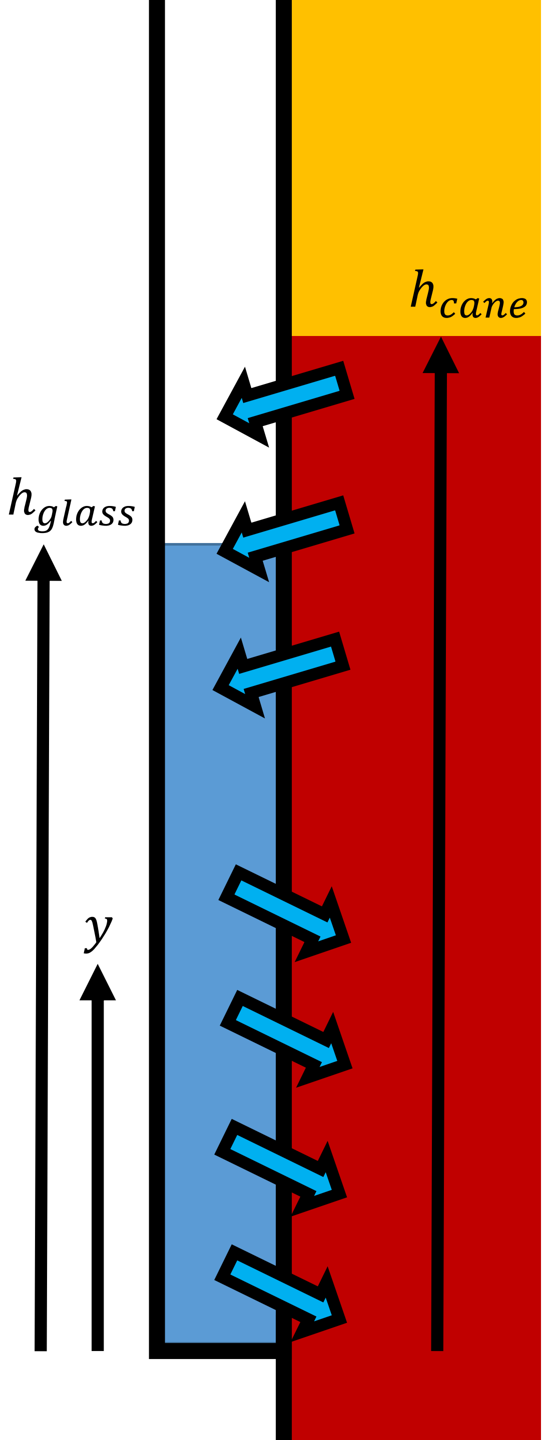

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- This gives that:

$$h_{glass} = h_{cane}\sqrt{\gamma}$$

- But this overestimates the value of h_{cane}, predicting overflow

- This implies that the cause for the gap water is not dominated by hydrostatic pressure as we expected

A Porous Medium Model

- If the sugar cane is more compact and/or the fluid is more viscous then an alternative model is to treat the cane as a porous medium in which the (Darcy) velocity $\boldsymbol{q}$ is governed by Darcy's law,

$$\nabla \cdot \boldsymbol{q} = 0, \quad \boldsymbol{q} = -\kappa \, \nabla \, (\rho \, g \, y + p),$$

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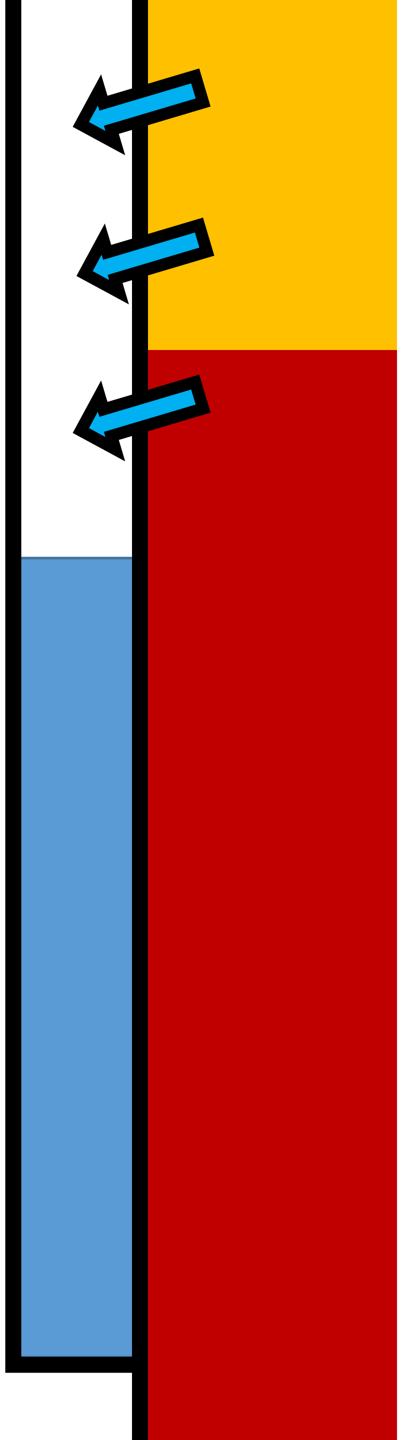
- We consider lateral flow ("seepage") driven by diffusion of water.

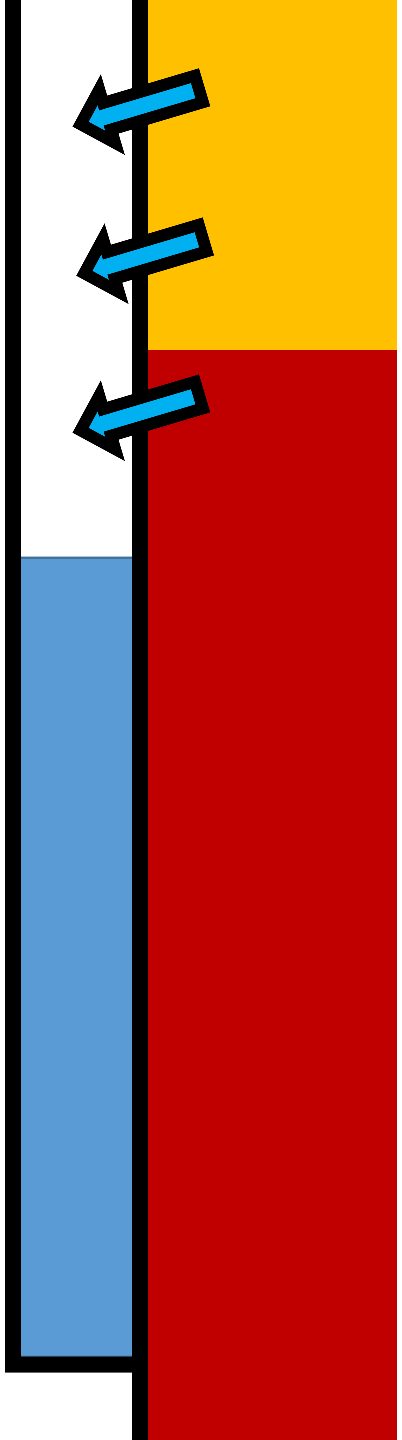


Diffusion of Water into the Air Gap

- Fick’s Law states that the flux of water is given by

$$f = -D \nabla \theta,$$

where D is the diffusivity and θ is the volume fraction of water.



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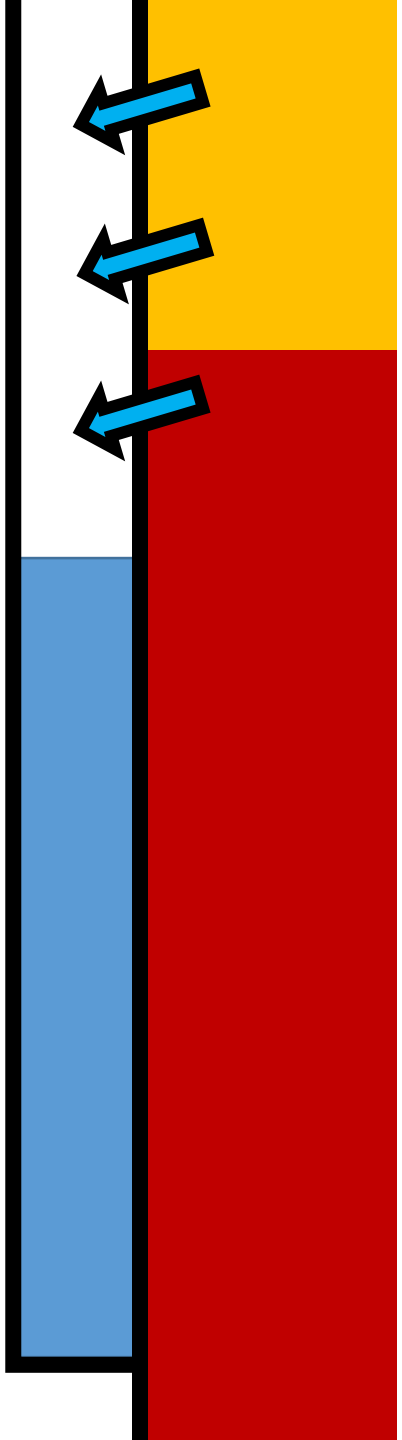

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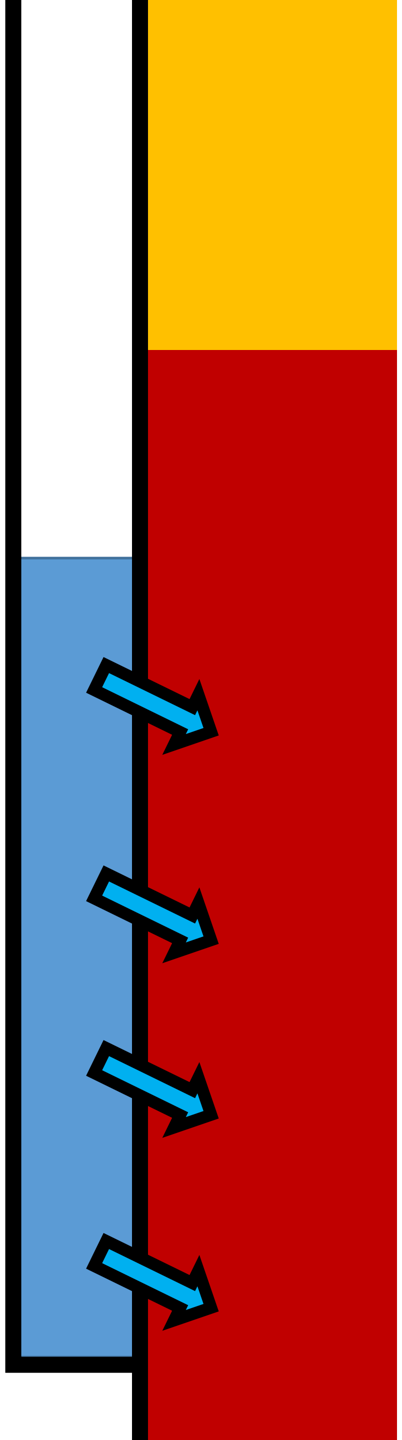
- This can be solved exactly for $\theta(x,t)$ and predicts that the total flux per unit width into the air gap is

$$\gamma\sqrt{\frac{D}{T}} \times \text{Area of Seepage Face} = \gamma\sqrt{\frac{D}{T}}\left(h_{\text{glass}} - h_{\text{cane}}\right),$$

where T is the time for the cane to transit the window (around 12 seconds).



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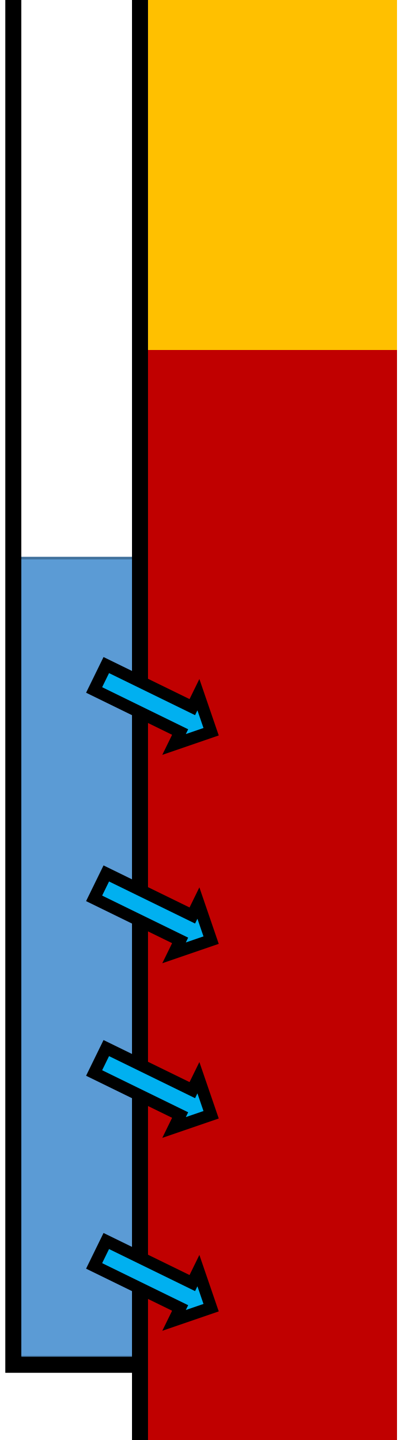

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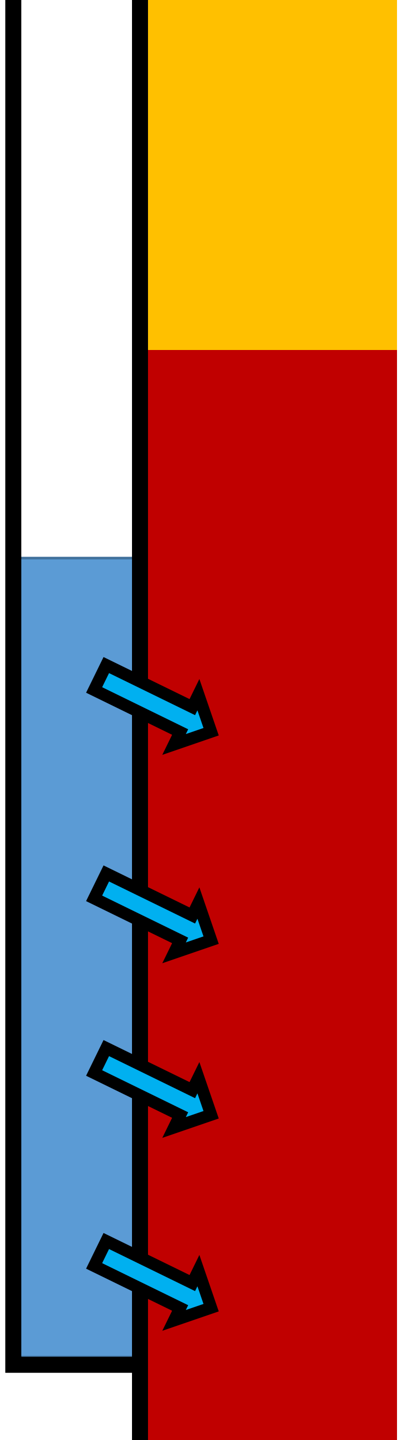
- A possible model for this flow is that the flux per unit width into the sugar cane is

$$\frac{K\,\rho\,g\,h_{\text{glass}}}{L} \times \text{Submerged Area} = \frac{K\,\rho\,g\,{h_{\text{glass}}}^{2}}{L},$$

where L is the penetration depth into the sugar cane.



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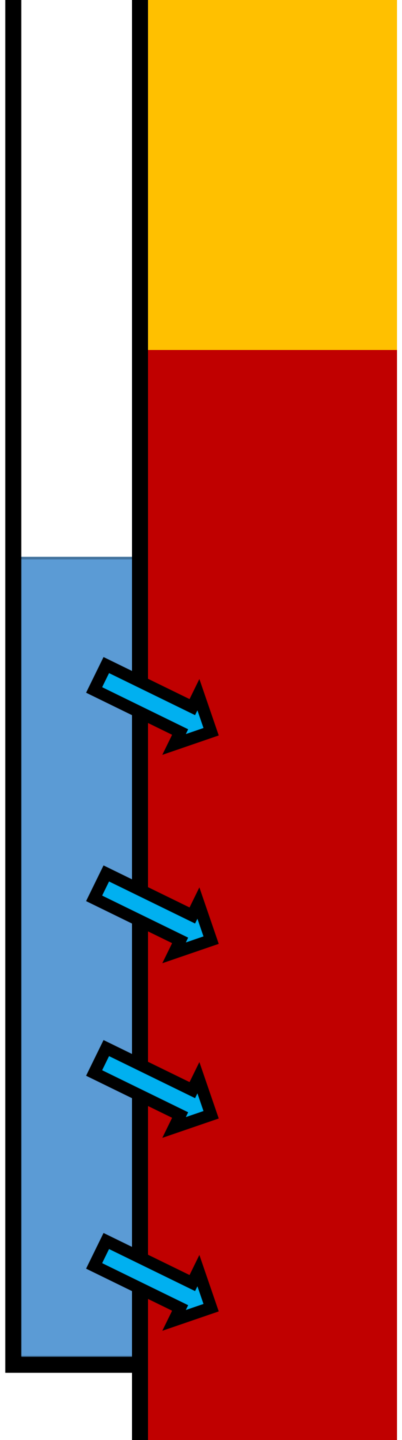

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- A thin-layer version of this calculation confirm these estimates.

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- For realistic numbers this (very roughly!) gives

$$h_{cane} - h_{glass} \simeq h_{glass},$$

which is, rather disappointingly, is somewhat worse than the corresponding prediction of the simple model.

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- Full (probably numerical) solutions of both the diffusion-driven seepage and pressure-driven outflow problems are necessary to confirm (or disprove) the order of magnitude estimates.

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- However, this also predicts a glass water level that is too low
- Some combination of these mechanisms or other effects are clearly at play here and further experiments are required to pin-point the cause